Low-frequency anomalies and scaling of the dynamic conductivity in the quantum Hall effect

B. M. Gammel and W. Brenig
Physik Department der Technischen Universität München, T30, 85747 Garching, Germany
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A numerical study of the dynamic conductivity $\sigma_{xx}(\omega)$ in the lowest Landau level for a quantum Hall system with short-range and long-range disorder potentials is performed. In the latter case two distinct types of low-frequency anomalies are observed: a scaling regime with an anomalous diffusion exponent of $\eta=0.36\pm0.06$ independent of the potential correlation range λ and a semiclassical regime giving evidence of the existence of long time tails in the velocity correlation decaying proportional to t^{-2}. The range of validity of this behavior increases with increasing λ. The universal value of the critical conductivity is $\sigma_{xx}^c=(0.5\pm0.02)e^2/h$ for $\lambda=0$ to 2 magnetic lengths. [S0163-1829(96)00720-5]

The localization-delocalization phase transition which occurs between two quantum Hall steps has attracted much attention recently. In the transition region novel scaling features showed up in the behavior of the localization length,[1–5] in the spatial structure of the wave function,[6–8] in the low-frequency response,[9–12] and in the static conductivity.[5,13,14] The importance lies in the universality of most features with respect to the stochastic impurity potential or the Landau level.

The localization length diverges as $|E-E^c|^{-\nu}$ at a critical energy in the center of each Landau level, where $\nu=2.35\pm0.03$ is independent of the stochastic impurity potential[3–5,14,15] and the Landau level.[16] The critical dissipative conductivity $\sigma_{xx}^c=e^2/2h$ at E^c is claimed to be universal, irrespective of the range of the potential[13,15,17] and also within a semiclassical approximation.[11] The scaling behavior of the dissipative conductivity σ_{xx} close to the critical point is governed by ν and the generalized fractal dimension D_2.[14] In an important experiment Koch *et al.*[18] were able to determine the critical exponent ν directly by studying the scaling of the peak width of the diagonal resistance ρ_{xx} as a function of the system size. The obtained value of $\nu=2.3\pm0.1$ for the lowest three Landau levels agrees well with theoretical results.

In this paper we present a numerical study of the dynamic conductivity $\sigma_{xx}(\omega)$ in the lowest Landau level for short-range and long-range impurity potentials. It is found that in the latter case there are two distinct regimes: a scaling regime that is characterized by power law scaling of the conductivity with an exponent related to η (Ref. 9) and a semiclassical regime giving evidence of the existence of t^{-2} long time tails in the velocity correlation function which show up in a *linear* decrease of the dynamic conductivity from the fixed point value $\sigma_{xx}^c=e^2/2h$. Furthermore, we show that η and σ_{xx}^c are independent of the range of the potential correlations.

In a previous paper[14] we developed a recursive Green function method for a direct numerical calculation of the dissipative part of the dynamic conductivity $\sigma_{xx}(L,\omega,\varepsilon)$ as a function of the frequency ω and the system width L. We introduced an imaginary part ε of the frequency as an important control parameter for the thermodynamic limiting process in finite systems. The thermodynamic limit is achieved by increasing the system size $L\to\infty$ and finally decreasing $\varepsilon\to0^+$, in order to retain all contributions from the spectrum of the Hamiltonian. The method has been applied to analyze the scaling relation of the static conductivity.

In the following it will be used to explore the scaling properties of the dynamic conductivity for short-range and long-range random impurity potentials, which are characterized by the correlation function

$$\overline{V(\boldsymbol{r}+\boldsymbol{b})V(\boldsymbol{r})}=\frac{u^2}{2\pi\lambda^2}e^{-b^2/2\lambda^2}.$$

The latter case is of special interest since many experiments are performed with high mobility heterojunctions where the characteristic length of the potential fluctuations λ is large compared to the magnetic length l. If the two-dimensional electron gas is separated from the randomly placed donors by an undoped spacer of width d, the potential correlation length is of the order of d.

Chalker and co-workers[9,10] have shown that the wave function under quantum Hall conditions shows anomalous diffusive behavior at the transition point. In the hydrodynamic limit, $\omega\to0$, $q\to0$, the spectral function $S(\boldsymbol{q},\omega)$ obeys the general scaling ansatz

$$S(\boldsymbol{q},\omega)=q^{-2+\tilde{\eta}}\chi(\omega/q^z). \quad (1)$$

The dynamical exponent is $z=2$ for a system without Coulomb interaction.[20,21] The property of anomalous diffusion arises from the power law behavior of the scaling function χ, which vanishes like

$$\chi(\omega/q^z)\propto(\omega/q^z)^{-\eta/2} \quad (2)$$

for $\omega/q^z<1/\hbar c\rho(\epsilon_F)$ (c is a numerical constant). The exponent $\tilde{\eta}$ related to the static susceptibility is zero[19] and the value of $\eta=0.38\pm0.04$ has been obtained numerically for electrons in the lowest Landau level moving in a Gaussian white noise potential.[9] η is related to the generalized fractal dimension D_2 of the wave function by $\eta=2-D_2$.[7,10,19]

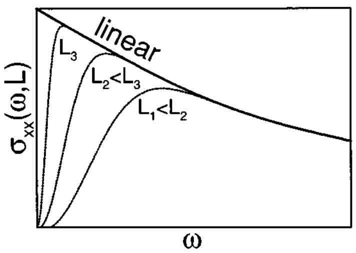

FIG. 1. Schematic illustration of the size and frequency dependence of $\sigma_{xx}(L,\omega)$ for a long-range potential. Increasing system width L is indicated by $L_1 < L_2 < L_3$.

In the case of $\omega/q^z > 1/\hbar c\rho(\epsilon_F)$, the regime of normal diffusion is characterized by $S(\boldsymbol{q},\omega) \propto (q^2/\omega^2)D$ and a diffusion constant D.

The dissipative part of the current correlation function $\Phi''(\boldsymbol{q},\omega)$ is related to $S(\boldsymbol{q},\omega)$ by

$$\Phi''(\boldsymbol{q},\omega) = (\omega^2/q^2)S(\boldsymbol{q},\omega). \tag{3}$$

In a finite sample the system size L provides the lower cutoff for the wave vector q. Thus the q dependence of $\Phi''(\boldsymbol{q},\omega)$ is reflected by the dependence of the conductivity $\sigma_{xx}(L,\omega)$ of L and can be obtained from a finite size scaling analysis.

In the following we will show that in the case of long-range potentials and $\omega/q^z > 1/\hbar c\rho(\epsilon_F)$ a nontrivial frequency dependence of the homogeneous conductivity appears—in contrast to the behavior in short-range potentials. The schematic illustration in Fig. 1 depicts the size and frequency dependence of $\sigma_{xx}(L,\omega)$ for a long-range potential. In a system of width L anomalous diffusive behavior with power law scaling is found below a frequency ω_L. With increasing system size $(L_1 < L_2 < L_3)$ the conductivity gap closes with the same power law as in short-range potentials. At sufficiently large L the conductivity maximum approaches $0.5e^2/h$ and a linear decay of $\sigma_{xx}(L\to\infty,\omega)$ becomes visible for small $\omega > \omega_L$. This is in contrast to conventional Drude-like behavior with a quadratic decay and indicates the existence of t^{-2} long time tails in the velocity correlations. As pointed out in the discussion below, this agrees with semiclassical results.[11]

As in our previous paper, the random Landau matrix model[3,22] for the generation of matrix elements with finite correlation length λ is used (lengths are given in units of the magnetic length l).

For short-range potentials, $\lambda = 0$, the dynamic conductivity $\sigma_{xx}(L,\omega)$ in the center of the lowest Landau band has been calculated for system sizes $L = 15$ to $L = 60$ with different values for ε as shown in Fig. 2. The frequency has been varied in the range $3\times10^{-5} < \hbar\omega < 0.01$. The energy scale is normalized to the second moment of the density of states $\rho(E)$, which is given by Wegner's function for $\lambda = 0$ and almost perfectly Gaussian for $\lambda = 2$.[14] Between 10^5 and 2×10^5 iterations were performed for each data point. The statistical errors are less than the symbol size.

The estimate for the exponent can be improved significantly if we use the fact that for small frequencies ω a finite value of ε causes a small and almost constant contribution to the real part of the dissipative conductivity. To obtain highly accurate results for the scaling behavior at low frequencies we eliminate this effect by fitting a power law with offset for all curves simultaneously. This method has been used successfully in Ref. 14. The results for the adjusted values of $\sigma_{xx}(\omega L^2)$ as a function of ωL^2 are shown in the double logarithmic plot Fig. 2. The data collapse onto a single curve within almost two orders of magnitude and we obtain the power law

$$\sigma_{xx}(\omega L^2) = c(\omega L^2)^{\eta''},$$

with $\eta'' = 1.82 \pm 0.03$ and $c \approx 0.07$ resulting in $\eta = 0.36 \pm 0.06$ $[\eta'' = 2 - \eta/2$ from Eqs. (1–3)]. The critical conductivity $\sigma_{xx}^c = 0.50 \pm 0.02$ is attained after a transition region within $1 < \omega L^2 < 10$. Above this value the conductivity does not depend on the system size any more and shows almost zero slope [with increasing frequency $\sigma_{xx}(\omega)$ decreases in Drude-like fashion, i.e., with a quadratic term]. This behavior is in contrast to the decay in long-range potentials which will be discussed in the following.

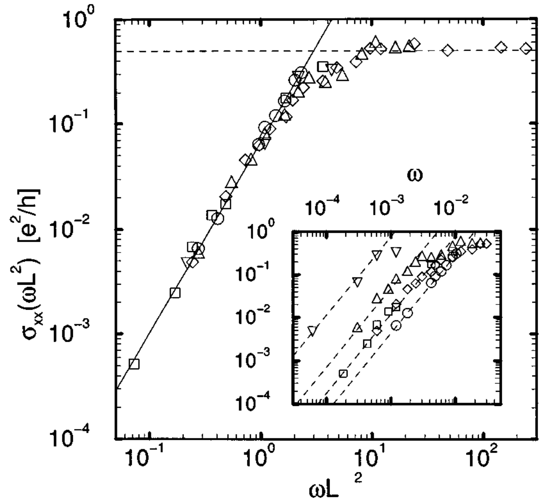

FIG. 2. $\ln\sigma_{xx}(\omega L^2)$, the logarithm of the dynamic conductivity in units of e^2/h vs $\ln\omega L^2$ for short-range potential $(\lambda = 0)$ with $L = 15$, $\varepsilon = 1\times10^{-4}$ (○), $L = 20$, $\varepsilon = 5\times10^{-5}$ (◇), $L = 20$, $\varepsilon = 1\times10^{-5}$ (□), $L = 30$, $\varepsilon = 2\times10^{-5}$ (△), and $L = 60$, $\varepsilon = 5\times10^{-6}$ (▽). The solid line indicates the power law with $\eta'' = 1.82 \pm 0.03$ and the broken line is $\sigma_{xx}^c = 0.5$. The inset shows the unscaled data as a function of ω.

At first the scaling regime below $\omega < \omega_L$ will be examined for a long-range potential with $\lambda = 2$. The exponent η has been calculated the same way as in the case of short-range potentials. In Fig. 3 it can be seen that the conductivities for various system sizes and frequencies again collapse onto a power law. The value obtained for the exponent is $\eta'' = 1.80 \pm 0.06$. Thus we can conclude that within the given error the exponent η is not affected by the range of the potential correlations.

For larger frequencies, $\omega > \omega_L$, the dynamic conductivity shows semiclassical behavior. In Fig. 4 the conductivity $\sigma_{xx}(L,\omega)$ is plotted versus frequency ω for a system of width $L = 30$. In order to avoid artifacts which result from large values of ε, the calculations have been performed for two very small values $\varepsilon = 1\times10^{-4}$ and $\varepsilon = 1\times10^{-5}$. In the latter case 5×10^5 iterations have been performed for each

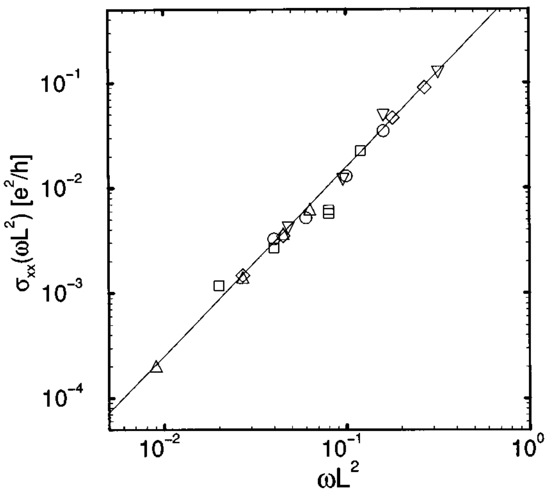

FIG. 3. $\ln\sigma_{xx}(\omega L^2)$, the logarithm of the dynamic conductivity in units of e^2/h vs $\ln\omega L^2$ for a long-range potential, $\lambda=2$, with $L=20$, $\varepsilon=1\times10^{-5}$ (○), $L=20$, $\varepsilon=3\times10^{-6}$ (□), $L=30$, $\varepsilon=1\times10^{-5}$ (◇), $L=30$, $\varepsilon=1\times10^{-6}$ (△), and $L=40$, $\varepsilon=1\times10^{-6}$ (▽). The solid line indicates the power law with $\eta''=1.80\pm0.06$.

value of ω and the results of 3–5 systems have been averaged to decrease the statistical errors further (indicated by error bars). Above the scaling regime ($\omega_L>0.013$) the linear decay of $\sigma_{xx}(\omega)$ is evident. It can be seen that an increase of ε causes an almost constant additional contribution to $\sigma_{xx}(\omega)$ and does not affect the power law of the decay. A significantly larger value of ε would smooth out the linear behavior to a Drude-like form. For comparison $\sigma_{xx}(\omega)$ has also been calculated for an intermediate correlation length $\lambda=1$. The linear decay is still present, but the slope is smaller. (The curve is somewhat shifted upwards due to a larger value of the imaginary frequency part ε.)

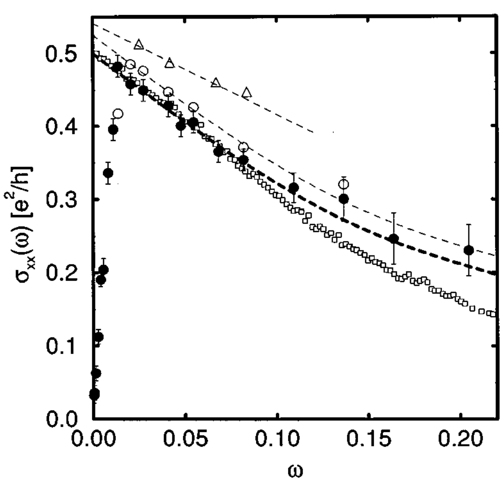

FIG. 4. $\sigma_{xx}(\omega)$, the dynamic conductivity in units of e^2/h vs ω for a system of width $L=30$ and a long-range potential with $\lambda=2$, $\varepsilon=1\times10^{-5}$ (●), and $\varepsilon=1\times10^{-4}$ (○). For comparison the result for intermediate range $\lambda=1$, $\varepsilon=1\times10^{-4}$ (△), and the semiclassical result (Ref. 11) is drawn (□) . The broken lines serve as a guide to the eye.

The critical conductivity of $\sigma_{xx}^c=(0.50\pm0.01)e^2/h$ can be obtained from the $\varepsilon=10^{-5}$ data by a linear extrapolation to $\omega=0$. It is equal to the value obtained for the short-range potential and in accordance with the universal value to a high precision.

In the high field limit, $l\ll\lambda$, the impurity potential becomes smooth on the scale of the magnetic length and the electrons move along the equipotential lines of the impurity potential. Within a semiclassical model Evers and Brenig[11] obtained the frequency dependence of the dissipative conductivity $\sigma_{xx}(\omega)$ and a finite value for the static conductivity at the transition point. Amazingly the semiclassical model yields the same universal critical value $\sigma_{xx}(\omega\to0)=(0.50\pm0.02)e^2/h$ as the full quantum problem and a significant deviation from a Drude-like behavior. For finite but small ω one obtains

$$\sigma_{xx}(\omega)=\sigma_{xx}^c-\text{const}|\omega|. \tag{4}$$

This result even suggests a cusp at $\omega=0$. Thus the averaged velocity correlation function $\Phi(t)=\langle v_x(t)v_x(0)\rangle$ exhibits a t^{-2} long time tail. Here we have derived this result by directly evaluating the Kubo formula. This way we have shown that long time tails are stable under inclusion of quantum mechanical interference effects depending on the range of the correlations in the potential.

Comparing our results with the semiclassical model[11] (Fig. 4, squares), we can argue now that the motion of electrons on equipotential curves is indeed a good picture for smooth potentials with a correlation length of approximately $\lambda>2$. The dynamic conductivity in the semiclassical regime, $\omega>\omega_L$, is mainly determined by the decay of the velocity correlations of electrons moving classically on equipotential lines.

Indications that non-Drude behavior might also be present, if one takes into account quantum mechanical propagation in long-range potentials, have been given in a split operator wave packet study,[12] although a critical conductivity of less than 40% of the universal value was found. Also a close reinspection of the numerical data of Ando[23] for the long-range samples shows such a deviation from Drude-like behavior although with large statistical errors.

Since up to now there is no analytic theory for the integer quantum Hall effect that could make predictions about the behavior of the dynamic conductivity $\sigma_{xx}(\omega)$, it would be desirable to investigate the low-frequency response experimentally. It can be estimated that the linear decay should show up at frequencies in the MHz to GHz range. Temperatures in the mK range are necessary to avoid inelastic scattering and the softening of the Fermi edge. Both effects would result in a Drude-like behavior that could hide the linear decay.

In conclusion, we have unambiguously shown that long time tails in the velocity correlations are present in a system under quantum Hall conditions (interactions neglected). Thus we predict a measurable deviation in the low-frequency behavior of the dynamic conductivity from conventional Drude-like behavior. Furthermore, we find that σ_{xx}^c at the center of the lowest Landau level is equal to $(0.50\pm 0.02)e^2/h$ for short-range and long-range potentials, in excellent agreement with the hypothesis of universality. Also the exponent of anomalous diffusion $\eta = 0.36\pm 0.06$ is shown to be independent of the potential correlation length within the investigated range.

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