Comment on "Analytic Structure of One-Dimensional Localization Theory: Re-Examining Mott's Law"

In a recent Letter A. O. Gogolin [1] has challenged the established point of view that Mott's prediction for the dynamical conductivity of a localized electron system is correct. The intuitive argument [2] leads in one dimension to a dynamical conductivity of the form $\omega^2 \ln^2 \omega$. Later, the precise, asymptotical result

$$\Re\, \frac{\sigma(\omega)}{\sigma_0} = \nu^2 \big(\ln^2 \nu - \frac{\pi^2}{4} + (2\mathcal{C} - 3) \ln \nu - C + \cdots \big) \quad (1)$$

($\nu = 2\omega\tau$, $\mathcal{C}$ denotes the Euler-Mascheroni constant $0.5772\ldots$ and $\sigma_0 = e^2 v_F \tau/\pi$ per spin). has been derived by several authors using different methods, see e. g. Refs. [3–5]. (We are not aware of any analytical prediction for the constant C.) Gogolin presents a purely formal calculation which yields

$$\Re\, \frac{\sigma(\omega)}{\sigma_0} = \frac{1}{3}\, \nu^2 \big(\ln^3 \frac{1}{\nu} + \cdots \big) \quad (2)$$

(eq. (22) in Ref. [1] with $\sigma_0 = 4$). In view of the mentioned variety of works corroborating Mott's conclusion this is quite unexpected. If Gogolin were right, then one of the thought to be most profound chapters in localization theory would have to be rewritten. In fact, however, as we will demonstrate below, he is not.

Gogolin's analysis starts from the famous recursion equations derived first by Berezinskii [3]. The equations can be solved in a standard manner by mapping them to a differential equation. Gogolin's claim is that the previous solution of this equation is incorrect and hence also the conductivity law $\omega^2 \ln^2 \omega$ derived thereof. He argues that previous authors have not properly taken into account discreteness of the spectrum of the equation.

A simple method to check Berezinskii's result is to solve the recursion equations for the conductivity numerically. (For details see Ref. [6].) The algorithm is very stable and has been used down to frequencies $\nu = 5 \cdot 10^{-6}$ where $M = 10^8$ in a calculation with 40 digits (fixed) precision. For even larger $M = 2 \cdot 10^8$ or more digits, e.g. 60, σ does not change implying that rounding errors are irrelevant. Fig. 1 shows our result. The agreement of the numerical data with the Mott/Berezinskii-solution is perfect over more than 3 decades while the data is completely incompatible with Gogolin's $\ln^3 \omega$ term.

One may ask where Gogolin's approach fails. We believe that the problem stems from the "leading logarithmic approximation", the only step in the calculation which is not exact. The expression eq. (19) in Gogolin's paper derived within this approximation may be

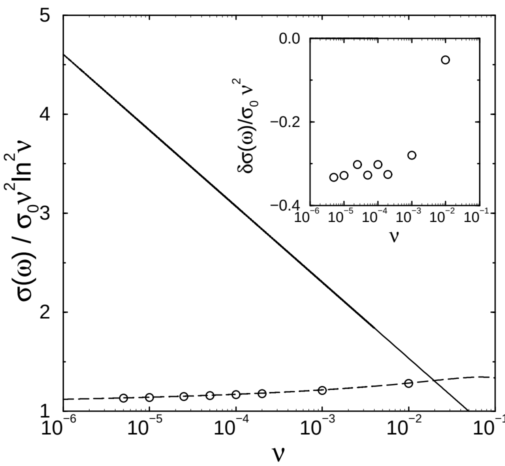

FIG. 1. Dynamical conductivity from solving the Berezinskii recursion equations ($\nu = 2\omega\tau$). Numerical solution (○), Berezinskii's solution, eq. (1) (dashed), Gogolin's result, eq. (2) (solid). Inset: Determining C by subtracting first three terms in eq. (1) from numerical data: $C \approx 0.3$.

sufficient for obtaining the leading term $\propto i\omega$. However, the real part of σ is of higher order in ω and presumably to this order corrections exist that have been ignored by Gogolin and that cancel the $\ln^3 \omega$ term. We also mention, that in contrast to Gogolin's statements the length $\ell \ln(1/\nu)$ has been identified and discussed in the literature as a relevant scale, e. g. in Ref. [7].

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